2.20.1. Semantic Problems:
Validity, Consistency, and Counterexample Sets

1. It was noted in 2.19 that any sentence follows validly from itself. Use **counterexample sets** and **(in)consistency** to explain why this is so.

2. It was also noted that, beginning with a valid argument, adding further premises always yields a (bigger) valid argument. For example, since the following familiar argument is valid,

$$\begin{array}{l} 1.\ (P \vee Q) \\ 2.\ \sim P \\ \hline \therefore Q \end{array}$$

this next argument is valid as well.

$$\begin{array}{l} 1.\ (P \vee Q) \\ 2.\ \sim P \\ 3.\ X \\ \hline \therefore Q \end{array}$$

Use **counterexample sets** and **(in)consistency** to explain why adding further premises always leaves the (larger) argument valid.

(*Hint: use the features of consistent and inconsistent sets discussed in* 2.17 §4)

3. It was also noted that an argument whose **conclusion** is a **tautology** is sure to be valid. (In other words: a tautology follows validly from any premise(s).) Use **counterexample sets** and **(in)consistency** to explain why this is so.

4. It was also noted that an argument with inconsistent premises is sure to be valid. Use **counterexample sets** and **(in)consistency** to explain why this is so.

5. Show that if an argument is invalid, then removing one or more premises will always yield a (smaller) argument which is also invalid.

> (*Hint: use the features of consistent and inconsistent sets discussed in* 2.17 §4)

6. Show that **adding a tautology** as an additional premise will **not affect** the **validity** of the argument – that is, if the original argument is invalid, adding a tautology as an additional premise will yield a (new, bigger) argument that's also invalid; and if the original argument is valid, adding a tautology as an additional premise will yield a (new, bigger) argument that's also valid.

> (*Hint: first show that adding a tautology won't affect the* (in)*consistency of a set. See also* 2.18.1 Problem A19)

7. The **(in)validity of an argument is not affected by adding an additional premise that is entailed by the existing premise(s)**. So, for example, since the left argument is invalid, the right one is too – because the added premise, “~P,” is entailed by the original premise “~(P ∨ Q)”.

INVALID	**INVALID**
1. ~ (P ∨ Q)	1. ~ (P ∨ Q)
	2. ~P
———	———
∴ Q	∴ Q

Use this principle (along with the results from Problem (3) above) to explain the result reported in Problem (6) above.

8. (a) 2.18.1 Problem C showed that a consistent sentence never entails a contradiction. (Equivalently: that **the only (sort of) sentence that entails a contradiction is a contradiction**.)

That means the following principle holds, for one-premise arguments.

> (1) **A contradiction follows validly only from a contradiction**.

Explain how Principle (1) can be strengthened to hold for arguments with any number of premises (so long as those sentences form an inconsistent set) – yielding Principle (2).

> **(2) A contradiction follows validly only from an inconsistent set of sentences**. (Equivalently: **no contradiction follows from a consistent set of sentences**.)

(b) Use the features of inconsistent premises and validity discussed in 2.20 to strengthen (2) into Principle (3).

> **(3) A contradiction follows validly from every inconsistent set of premises, and only from an inconsistent set of premises**.

9. Note that for each of the following **inconsistent** sets of sentences, the **negation of any one sentence** in the set **follows validly** from the remaining sentences.

Example 1: {(P ∨ Q), ~P, ~Q}

All the following arguments are valid:

(P ∨ Q), ~P ∴ ~~Q
(P ∨ Q), ~Q ∴ ~~P
~P, ~Q ∴ ~(P ∨ Q)

Example 2: {P, ~P, Q}

All the following arguments are valid:

P, ~P ∴ Q
P, Q ∴ ~~P
Q, ~P ∴ ~P

Example 3: {(P ∧ ~P), X}

All the following arguments are valid:

(P ∧ ~P) ∴ ~X
X ∴ ~(P ∧ ~P)

Appealing only to the defining features of **validity**, **validity counterexample**, and **inconsistency**, along with the semantics for **negations**, show that **any inconsistent set** can be translated into a valid argument in this way.

10. In 2.19.1 we noted that the following general form, though typically yielding invalid arguments when formal sentences fill the blanks, will in certain mutant, limit cases yield a valid argument.

1. (● ∨ ▲)
2. ~ ●

∴ ✱

Given the link noted in 2.20 between validity and inconsistency, make a similar argument that the following general logical form, while typically yielding consistent sentences when its blanks are filled by sentences, will yield an **inconsistent** sentence in certain unusual cases.

(● ∧ ▲)